
Rational Numbers

Rational Numbers: A rational number is defined as a number which can be expressed in the form of numerator/denominator, where numerator and denominator are integers and denominator is not equal to zero.

Example: $3/8, 7/9, -7/4$ etc

Positive and Negative Rational numbers: If the numerator and denominator both are positive or both are negative, then it is called positive rational numbers otherwise it is called negative rational numbers.

For Example: $6/7, -4/-3$ are positive rational numbers.

$-5/7, 1/-9$ are negative rational numbers.

Standard Form of a Rational Numbers:

A rational number is in its standard form if its denominator is positive and there is no common factor (except) 1 between the numerator and denominator.

For example: $6/7, -4/9$

Equivalent Rational number:

If in a rational number, we multiply the numerator and denominator by the same non-zero integer, we obtain another rational number which is equivalent rational number.

Example: $-2/3$

$$-2 \times 2 / 3 \times 2 = -4/6$$

$$-2 \times 3 / 3 \times 3 = -6/9$$

Absolute Value of a Rational Number:

The absolute value of a/b is written as $|a/b|$

Example: Find the absolute value of $-2/7$

Solution: $|-2/7|$

$$2/7$$

Properties of Rational Numbers:

1) Closure Property:

Addition: For rational numbers a and b , addition of a and b is also a rational number.

Example: $1/2 + 1/3$

$$(3+2)/6$$

$$= 5/6$$

Closure property holds true for addition of rational numbers.

Subtraction: For rational number a and b , subtraction of $a-b$ is also a rational number.

Example: $1/2 - 5/3$

$$(3-10)/6$$

$$-7/6$$

Closure Property holds true for subtraction of rational numbers.

Multiplication: For two rational numbers a and b , multiplication of a and b is also rational number.

Example: $\frac{4}{7} \times \frac{3}{2}$

$$= \frac{12}{14}$$

$$= \frac{6}{7}$$

Closure Property holds true for multiplication of rational numbers.

Division: For rational numbers a and b, division - $\frac{a}{b}$ is also a rational number.

Example: $(\frac{4}{6})/(\frac{1}{6})$

$$= \frac{24}{5}$$

Closure Property holds true for division of rational numbers.

2) Commutative Property

Addition: For rational number a and b

$$a+b=b+a$$

Example: $\frac{2}{3} + \frac{4}{5}$

$$\frac{2}{3} + \frac{4}{5} = \frac{4}{5} + \frac{2}{3}$$

$$(\frac{10}{15} + \frac{12}{15}) = (\frac{12}{15} + \frac{10}{15})$$

$$\frac{22}{15} = \frac{22}{15}$$

Commutative Property holds true rational numbers.

Subtraction: For rational numbers a and b

$$a-b \text{ not equals to } b-a$$

Example: $\frac{6}{7} - \frac{4}{5}$

$$\frac{6}{7} - \frac{4}{5} \neq \frac{4}{5} - \frac{6}{7}$$

$$(\frac{30}{35} - \frac{28}{35}) \neq (\frac{28}{35} - \frac{30}{35})$$

$$\frac{2}{35} \neq -\frac{2}{35}$$

Commutative Property does not hold true for the subtraction of rational numbers.

Multiplication: For rational numbers a and b,

$$a \times b = b \times a$$

Example: $\frac{4}{7} \times \frac{3}{7}$

$$\frac{4}{7} \times \frac{3}{7} = \frac{3}{7} \times \frac{4}{7}$$

$$\frac{12}{49} = \frac{12}{49}$$

Division: For rational numbers a and b,

$$\frac{a}{b} \text{ is not equals } \frac{b}{a}$$

Example: $(\frac{4}{5})/(\frac{1}{3})$

$$(\frac{4}{5})/(\frac{1}{3}) \text{ is not equals to } (\frac{1}{3})/(\frac{4}{5})$$

$$\frac{12}{5} \text{ is not equals } \frac{5}{12}$$

Commutative Property doesn't hold for the division of rational numbers.

3) Associative Property:

Addition: For rational numbers a ,b and c,

$$a+(b+c)=(a+b)+c$$

Example: $1/3+(1/4+2/3)=(1/3+1/4)+2/3$

$$1/3+11/12=7/12+2/3$$

$$15/12=15/12$$

Subtraction:For rational numbers a,b and c,

$a-(b-c)$ is not equal to $(a-b)-c$

Multiplication:For rational numbers a,b,c,

$$a \times (b \times c) = (a \times b) \times c$$

Example: $1/2 \times (3/4 \times 5/6) = (1/2 \times 3/4) \times 5/6$

$$1/2 \times (15/24) = 3/8 \times 5/6$$

$$1/2 \times 5/8 = 5/48$$

$$5/16 = 5/16$$

Division:For rational numbers a,b and c,

$a/(b/c)$ is not equal to $(a/b)/c$

4)Distributive Property:

Addition:For rational numbers a,b and c,

$$a \times (b+c) = a \times b + a \times c$$

Subtraction:For rational numbers a,b and c

$$a \times (b-c) = a \times b - a \times c$$

Property of Zero Or Additive Identity:

When Zero is added to a rational number,the sum of rational number is same.

Example: $4/5$

$$= 4/5 + 0$$

$$= 4/5$$

Property of Multiplicative identity:

When 1 is multiplied by rational number, product is same as the rational number.

Example: $5/6$

$$= 5/6 \times 1$$

$$= 5/6$$